LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **STATISTICS**

THIRD SEMESTER – **NOVEMBER 2013**

ST 3506 – MATRIX AND LINEAR ALGEBRA

Date : 08/11/2013 Dept. No. Max. : 100 Marks
Time : 9:00 - 12:00

PART – A

Answer the following questions: **[10x2= 20]**

1. Define Skew-Symmetric matrix. Give an example.
2. Define trace of a square matrix.
3. Define singular and non-singular matrices.
4. Find the determinant $\begin{vmatrix} 1 & \log_y x \\ \log_x y & 1 \end{vmatrix}$.
5. When do we say that the vectors $X_1, X_2 \ldots, X_r$ are linearly dependent?
6. Explain linear homogeneous equations.
7. State any two properties of rank of a matrix.
8. Define 'Basis' of a vector space.
9. Define Characteristics roots of a matrix.
10. Determine the characteristic roots of the matrix M = $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.

PART – B

Answer any FIVE questions: **[5 x 8=40]**

11. If A and B commute, show that for every positive integer 'n',

$$(A + B)^n = \sum_{r=0}^{n} {}^nC_r\, A^r\, B^{n-r} .$$

12. Evaluate the determinant, and find its value when a+ b+ c =0.

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}$$

13. Show that the inverse of a symmetric matrix is symmetric.
14. Find the inverse of the matrix.

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$$

15. State and prove the necessary and sufficient condition for consistency of a system of linear equations.
16. Examine the consistency and solve

$$x + 2y - z = 3$$
$$3x + y + 2z = 1$$
$$2x - 2y + 3z = 2$$

17. Apply Laplace expansion using minors of the first two rows to find the determinant

$$\begin{vmatrix} 3 & 2 & 4 & 0 \\ 1 & 4 & 2 & 3 \\ 4 & 3 & 1 & 0 \\ 2 & 1 & 3 & 4 \end{vmatrix}.$$

18. State and prove Cayley-Hamilton Theorem.

PART - C

Answer any TWO questions: **[2 X20=40]**

19. (a) Show that every square matrix with complex elements can be expressed uniquely as the sum of a Hermitian and a Skew- Hermitian Matrix.

(b) Solve for x

$$\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0.$$

20. (a) Prove that

$$\begin{vmatrix} a & b+c & a^2 \\ b & c+a & b^2 \\ c & a+b & c^2 \end{vmatrix} = -(a+b+c)(a-b)(b-c)(c-a).$$

(b) Find the inverse of A by step-by-step reduction of $[A \vdots I]$ where

$$A = \begin{bmatrix} 2 & 3 & -1 \\ 0 & 1 & -1 \\ 2 & 1 & 2 \end{bmatrix}$$

21. (a) If $A \begin{bmatrix} 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & \cdots & 1 & 0 \\ \vdots & \vdots & & 0 & 0 \\ 0 & 1 & & 0 & 0 \\ 1 & 0 & & 0 & 0 \end{bmatrix}$ is an n x n matrix, show that det A = $(-1)^{\alpha}$ where $\alpha = n(n-1)/2$.

(b) Using Cramer's rule find the solution of

$$2x - y + 3z = 9$$
$$x + y + z = 6$$
$$x - y + z = 2$$

22. (a) If $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$ show that $|A| = |A_{11}|\,|A_{22}|$ if $A_{12} = \mathbf{0}$.

Also, show that, in general $|A| = |A_{11}|\,\left|A_{22} - A_{21}A_{11}^{-1}A_{12}\right|$.

(b) Determine the characteristic roots of the matrix

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}.$$

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